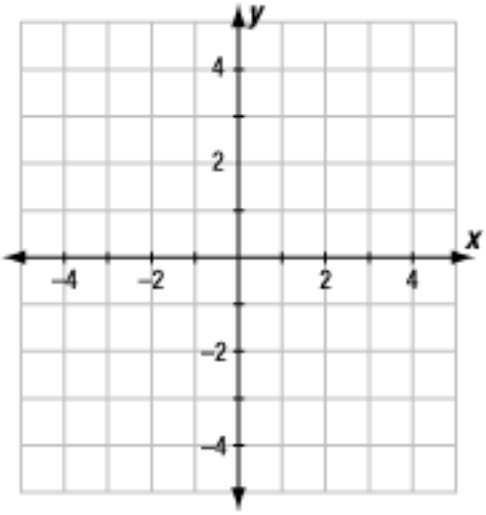
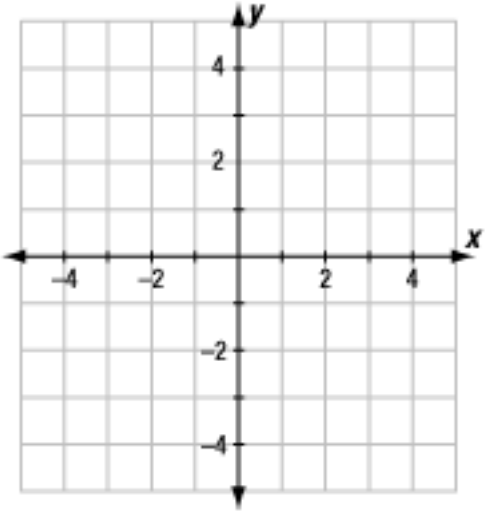
<u>Attendance Problems</u>. A figure has vertices *A, B,* and *C. After a transformation,* the image of the figure has vertices *A', B', and C'. Draw the* pre-image and the image on graph paper. Then identify the transformation.

1. A(-3, 1), B(-1, 1), C(-3, 4)
A′(3, 1), B′(5, 1), C′(3, 4)



2. A(2, 1), B(5, 1), C(4, 3)
A′(2, -1), B′(5, -1), C′(4, -3)



- I can draw, identify, and describe transformations in the coordinate plane.
- I can use properties of rigid motions to determine whether figures are congruent and to prove figures congruent.

Vocabulary		
dilation	Isometry	rigid transformation

Q: What does a foreign language teacher have in common with a geometry teacher?
A: They are both experts at translations.

Common Core

CC.9-12.G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the defintion of congruence n terms of rigid motions to decide if they are congruent.

CC.9-12.G.CO.7 Use the definition of congruence in terms of rigid motions to s show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

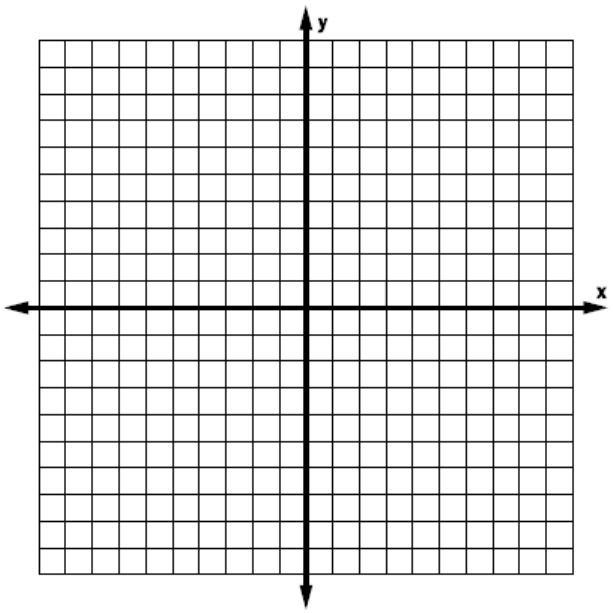
A **dilation** with scale factor k > 0 and center (0, 0) maps (x, y) to (kx, ky).

Remember!

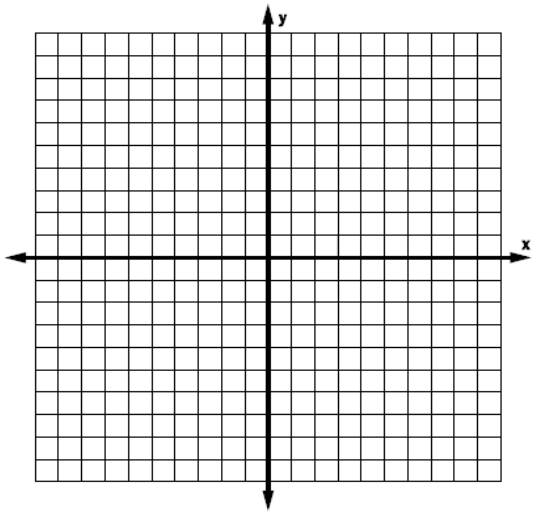
In a transformation, the original figure is the pre-image. The resulting figure is the image.

Video Example 1. Name the coordinates of the image points Identify and describe the transformation.

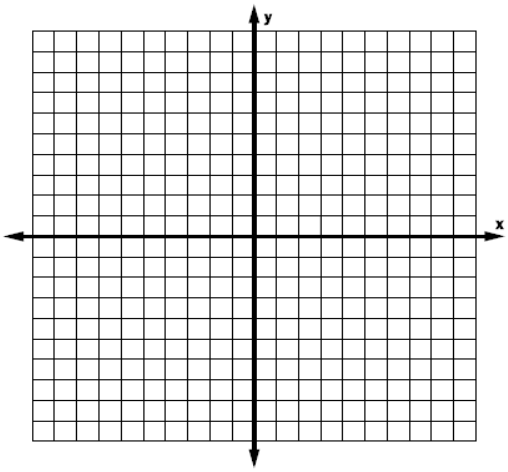
A. Apply the transformation M: (x, y) →(x - 3, y - 4) to the polygon with vertices P(1, 6), Q(1, 3), R(4, 3).



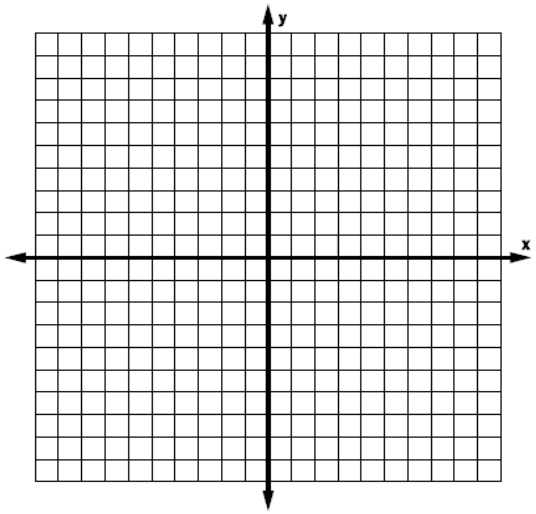
B. Apply the transformation $(x, y) \rightarrow (-x, y)$ to the polygon with vertices P(1, 6), Q(1, 3), & R(4, 3).



C. Apply the transformation $(x, y) \rightarrow (-y, x)$ to a polygon with vertices P(1, 6), Q(1, 3), R(4, 3).



D. Apply the transformation $(x, y) \rightarrow (3x, 3y)$ to a polygon with vertices A(2, 1), B(1, 1), C(1, 0), & D(-2, 0).

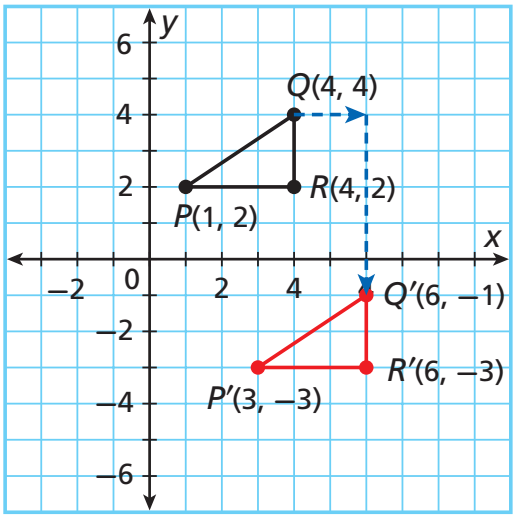


1 Drawing and Identifying Transformations

Apply the transformation M to the polygon with the given vertices. Identify and describe the transformation.

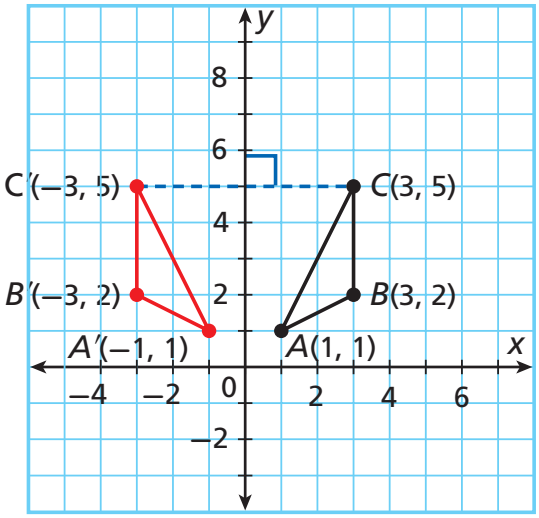
A $M: (x, y) \rightarrow (x + 2, y - 5)$
$P(1, 2)$, $Q(4, 4)$, $R(4, 2)$

This is a translation 2 units right and 5 units down.



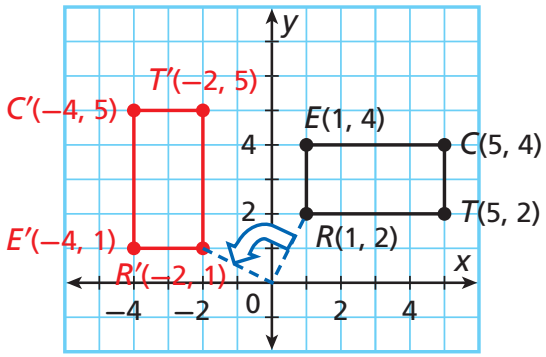
B $M: (x, y) \rightarrow (-x, y)$
$A(1, 1)$, $B(3, 2)$, $C(3, 5)$

This is a reflection across the y-axis.



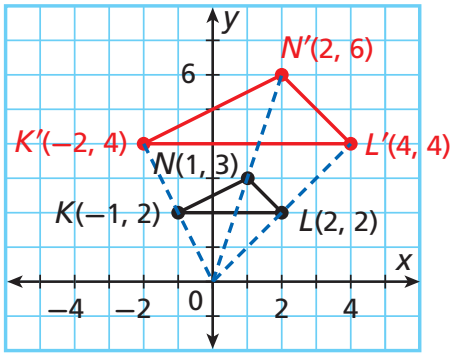
C $M: (x, y) \rightarrow (-y, x)$
$R(1, 2)$, $E(1, 4)$, $C(5, 4)$, $T(5, 2)$

This is a 90° rotation counterclockwise with center of rotation (0, 0).



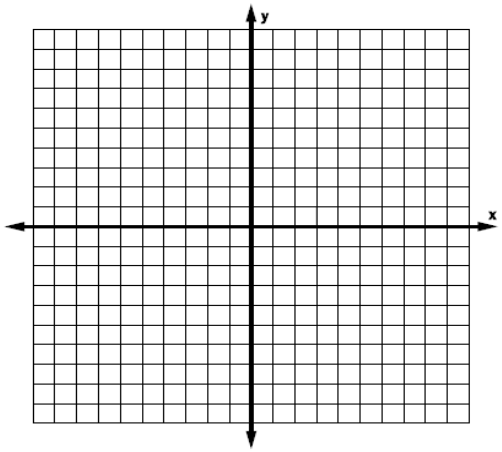
D $M: (x, y) \rightarrow (2x, 2y)$
$K(-1, 2)$, $L(2, 2)$, $N(1, 3)$

This is a dilation with scale factor 2 and center (0, 0).

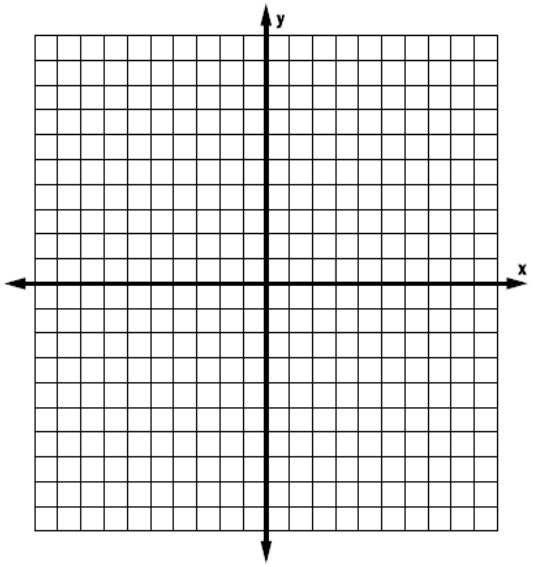


<u>Example 1</u>. Name the coordinates of the image points Identify and describe the transformation.

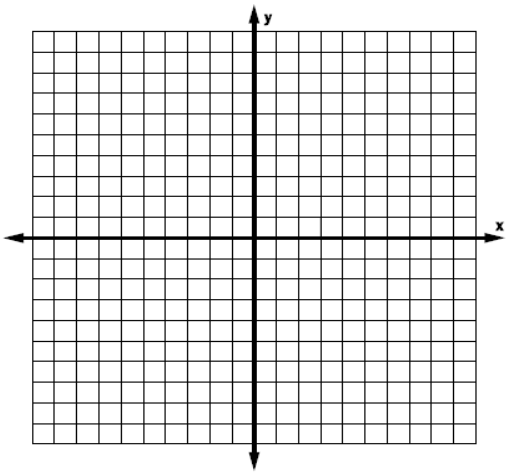
A. M: *(x, y) → (x - 4, y + 1)* to a polygon with vertices P(1, 3), Q(1, 1), R(4, 1).



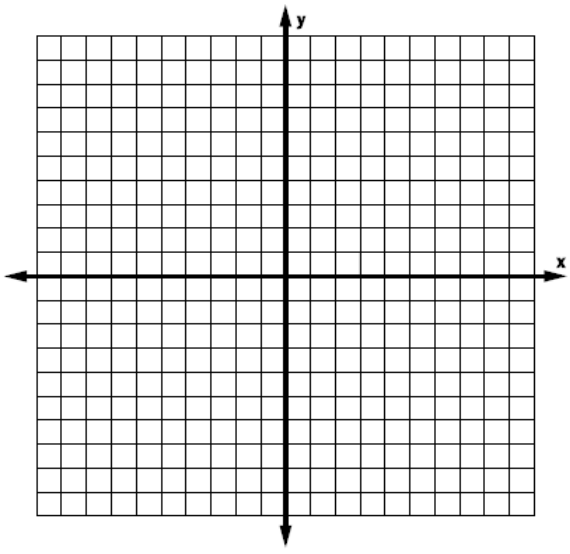
B. *M: (x, y) → (x, -y)* to a polygon with vertices A(1, 2), B(4, 2), C(3, 1).



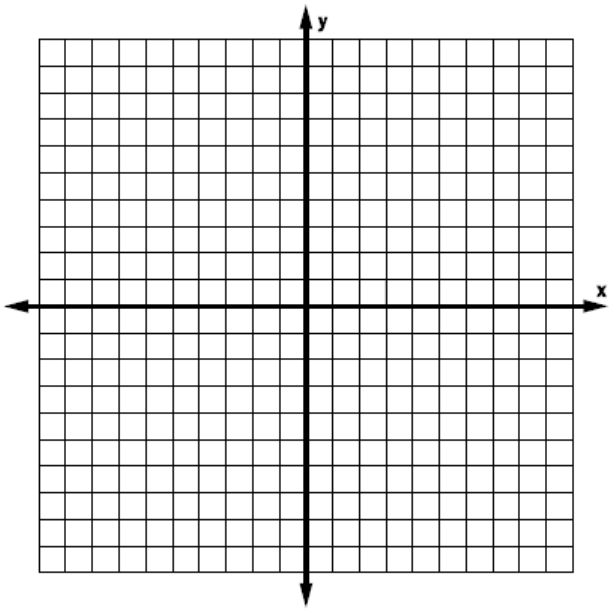
C. *M: (x, y) → (y, -x)* to a polygon with vertices R(-3, 0), E(-3, 3), C(-1, 3), T(-1,0).



D. *M: (x, y) → (3x, 3y)* to a polygon with vertices K(-2, -1), L(1, -1), N(1, -2).



9. <u>Guided Practice</u>. Apply the transformation M: $(x,y)\rightarrow(3x,3y)$ to the polygon with vertices D(1, 3), E(1, -2), and F(3, 0). Name the coordinates of the image points. Identify and describe the transformation.



Representing Transformations in the Coordinate Plane

TRANSFORMATION	COORDINATE MAPPING AND DESCRIPTION
Translation	$(x, y) \rightarrow (x + a, y + b)$ Translation a units horizontally and b units vertically
Reflection	$(x, y) \rightarrow (-x, y)$ Reflection across y-axis $(x, y) \rightarrow (x, -y)$ Reflection across x-axis
Rotation	$(x, y) \rightarrow (y, -x)$ Rotation about (0, 0), 90° clockwise $(x, y) \rightarrow (-y, x)$ Rotation about (0, 0), 90° counterclockwise $(x, y) \rightarrow (-x, -y)$ Rotation about (0, 0), 180°
Dilation	$(x, y) \rightarrow (kx, ky), k > 0$ Dilation with scale factor k and center (0, 0)

An ***<u>isometry</u>*** is a transformation that preserves length, angle measure, and area. Because of these properties, an isometry produces an image that is congruent to the pre-image.

A ***<u>rigid transformation</u>*** is another name for an isometry.

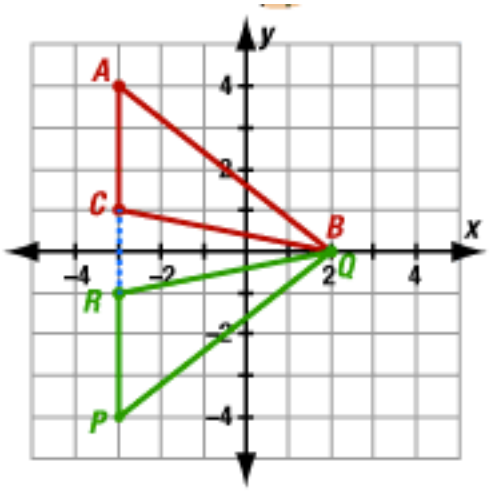
Transformations and Congruence

Translations, reflections, and rotations produce images that are congruent to their preimages.

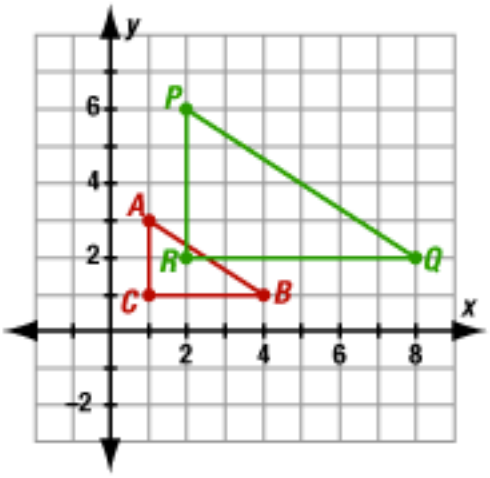
Dilations with scale factor $k \neq 1$ produce images that are not congruent to their preimages.

<u>Video Example 2</u>. Determine whether the polygons with the given vertices are congruent.

A. $A(-3,4),\ B(2,0), C(-3,1)$
$P(-3,-4), Q(2,0), R(-3,-1)$



B. $A(1,3), B(4,1), C(1,1)$
$P(2,6), Q(8,2), R(2,2)$



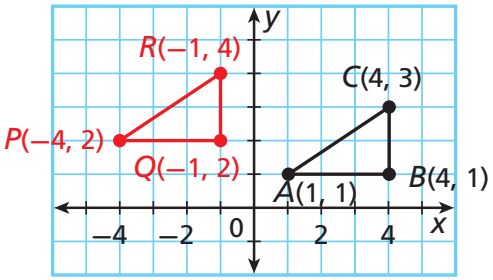
2 Determining Whether Figures are Congruent

Determine whether the polygons with the given vertices are congruent.

A $A(1, 1), B(4, 1), C(4, 3)$
$P(-4, 2), Q(-1, 2), R(-1, 4)$

The triangles are congruent because $\triangle ABC$ can be mapped to $\triangle PQR$ by a translation:

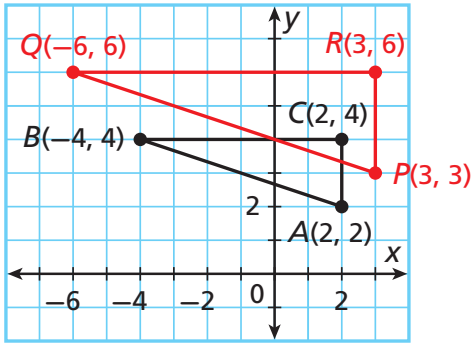
$(x, y) \rightarrow (x - 5, y + 1)$.



B $A(2, 2), B(-4, 4), C(2, 4)$
$P(3, 3), Q(-6, 6), R(3, 6)$

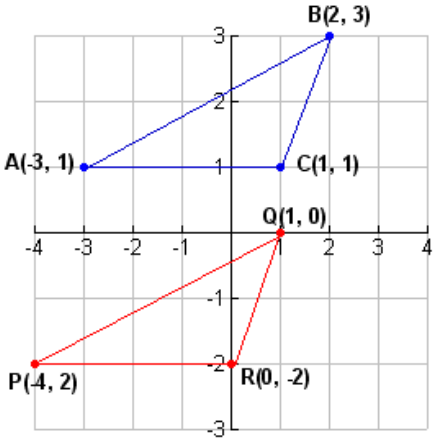
The triangles are not congruent because $\triangle ABC$ can be mapped to $\triangle PQR$ by a dilation with scale factor $k \neq 1$:

$(x, y) \rightarrow (1.5x, 1.5y)$.

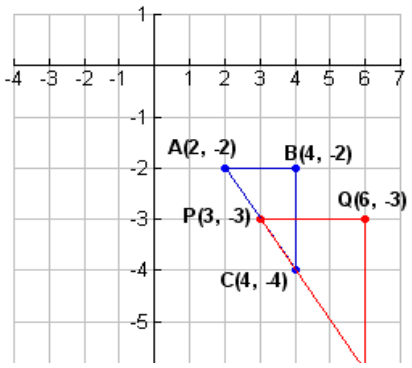


<u>Example 2</u>. Determine whether the polygons with the given vertices are congruent.

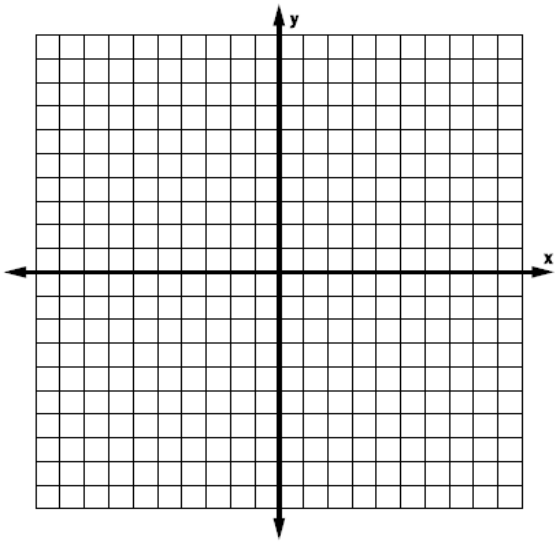
A. A(-3, 1), B(2, 3), C(1, 1)
P(-4, -2), Q(1, 0), R(0, -2)



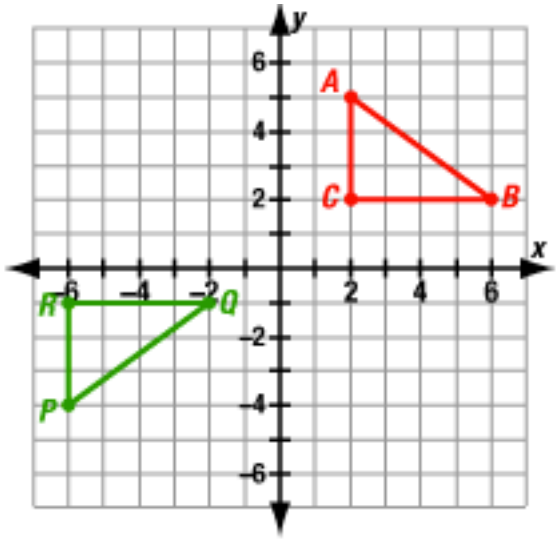
B. A(2, -2), B(4, -2), C(4, -4)
P(3, -3), Q(6, -3), R(6, -6).



10. <u>Guided Practice</u>. Determine whether the polygons with the given vertices are congruent. Support your answer by describing a transformation:
A(2, -1), B(3, 0), C(2, 3) and
P(1, 2), Q(0, 3), R(-3, 2).



<u>Video Example 3</u>. Prove that the polygons with the given vertices are congruent. $A(2,5), B(6,2), C(2,2)$
$P(-6,-4), Q(-2,-1), R(-6,-1)$

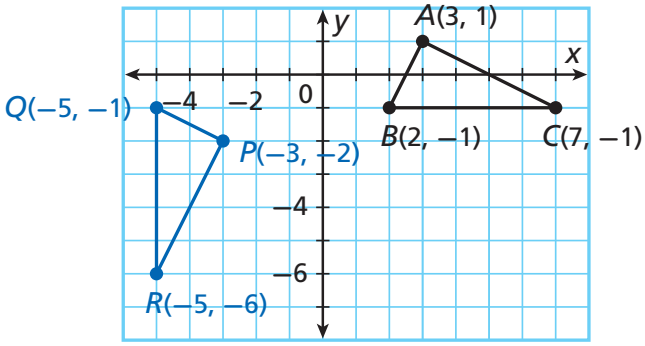


3 Applying Transformations

Prove that the polygons with the given vertices are congruent.

$A(3, 1)$, $B(2, -1)$, $C(7, -1)$
$P(-3, -2)$, $Q(-5, -1)$, $R(-5, -6)$

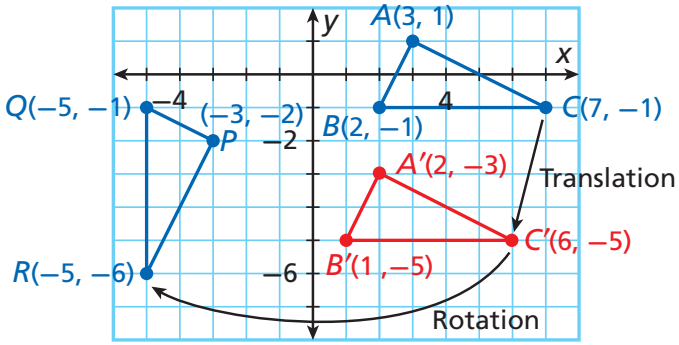
Graph the triangles. There is no apparent single transformation that maps $\triangle ABC$ to $\triangle PQR$. Look for a combination of congruence transformations that map $\triangle ABC$ to $\triangle PQR$.



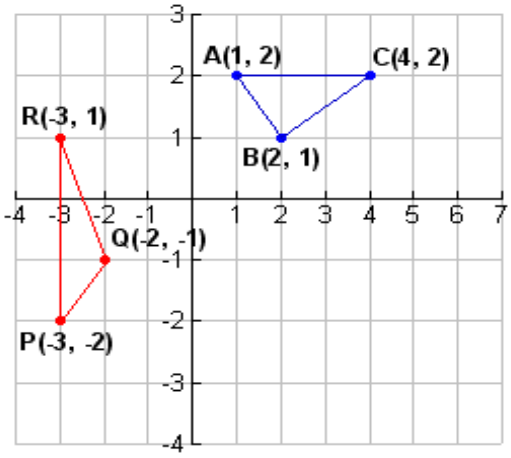
The triangles are congruent because $\triangle ABC$ can be mapped to $\triangle A'B'C'$ by a translation:

$(x, y) \rightarrow (x - 1, y - 4)$; and $\triangle A'B'C'$ can then be mapped to $\triangle PQR$ by a rotation:

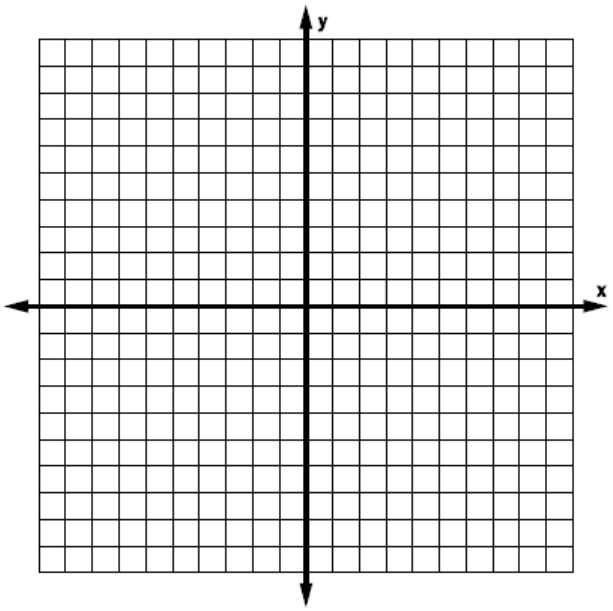
$(x, y) \rightarrow (y, -x)$.



Example 3. Prove that the polygons with the given vertices are congruent.
A(1, 2), B(2, 1), C(4, 2)
P(-3, -2), Q(-2, -1), R(-3, 1)



11. <u>Guided Practice</u>. Prove that the polygons with the given vertices are congruent: A(-4, -2), B(-2, 1), C(2, -2) and P(1, 0), Q(3, -3), R(3, 0).



Helpful Hint

Translations, reflections, and rotations can be called congruence transformations.

<u>4-1 Congruence and Transformations</u> *(p 220)* 15-23 odd, 24.

Video Example

4. What transformation is used to create the frieze pattern? Are sections of the frieze congruent? Explain your answer.

Library of Congress

4 ***Architecture Application***

What transformation is used to create the frieze pattern in this cast iron gate? Are sections of the gate congruent? Explain your answer.

Repeated horizontal translations create the frieze pattern. A translation of any section either to the left or to the right by a distance equal to the width of the section produces an image that is congruent to the preimage.

Example 4. Is there another transformation that can be used to create this frieze pattern? Explain your answer.

12. Guided Practice. Sketch a frieze pattern that can be produced by using reflections.

4-1 Congruence and Transformations *(p 220)* 15-23 odd, 24, 28, 30, 31, 33−37.